Available online at www.sciencedirect.com

SciVerse ScienceDirect

Procedia Engineering 38 (2012) 2601 – 2610

Procedia Engineering

www.elsevier.com/locate/procedia

INTERNATIONAL CONFERENCE ON MODELLING OPTIMISATION AND COMPUTING-(ICMOC-2012)

Dynamic Matrix Control of a Two Conical Tank Interacting Level System

V.R.Ravi[a], T.Thyagarajan[b], G.Uma Maheshwaran[c] a*

[a,c] *Dept. Of Electronics & Instrumentation Engineering,Velammal Engineering College,Chennai,India,*
[b] *Dept. Of Instrumentation Engineering, M.I.T Campus, Anna University Chennai, Chennai,India*

Abstract

The implementation of control algorithms for MIMO systems is often complicated due to variations in process dynamics that occur because of change in operating point and the characteristics of nonlinear dynamic coupling. Such difficulties often render the performance of industrial decoupling based decentralized PID controllers unsatisfactory. This work develops Dynamic Matrix Controller (DMC) which meets the design specifications for each loop independently for the entire nonlinear operating range. To verify the applicability of the developed DMC, this work presents a Two Conical Tank Interacting Level System (TCTILS) to examine the control performance. Simulation results show that the designed DMC realise a good dynamic behaviour of the TCTILS, a perfect level tracking with no overshoot, lesser settling time, reduced interaction and good rejection of external load disturbances.

Keywords: Dynamic Matrix Control; Two Conical Tank Interacting Level System; Model Predictive Control; Level control.

1. Introduction

Model Predictive Control (MPC) has established itself in industry as an important form of advanced multivariable control [1]. The formulation naturally handles time-delays, non linearity, multivariable interactions and constraints. Since the advent of MPC, various model predictive controllers had been evolved to address an array of control issues [2]. Dynamic Matrix Control (DMC) [3] is the most popular

* Corresponding author. Tel.: +91-9600181628; fax: +91-44-26591771.
E-mail address: vrravi_2007@yahoo.co.in.

1877-7058 © 2012 Published by Elsevier Ltd. Open access under CC BY-NC-ND license.
doi:10.1016/j.proeng.2012.06.306

Nomenclature

F_{IN1}	Input flow to Tank1
F_{IN2}	Input flow to Tank2
F_{OUT1}	Output flow from Tank1
F_{OUT2}	Output flow from Tank2
CV_1	Control valve at the inlet pipe of Tank1
CV_2	Control valve at the inlet pipe of Tank2
MV_1	Manual control valve at the outlet pipe of Tank1
MV_2	Manual control valve at the outlet pipe of Tank2
MV_{12}	Manual control valve at the intersection of Tank1 and Tank2
C	Cross sectional area of valves
a	Discharge coefficient
h_1	Liquid level in TANK1
h_2	Liquid level in TANK2
$A(h_1)$	Cross sectional area of conical tank at height h_1
$A(h_2)$	Cross sectional area of conical tank at height h_2
R	Maximum radius of conical tank
H	Maximum height of conical tank
P	Prediction horizon
M	Control horizon (number of controller output moves)
N	Model horizon (Process settling time in samples)
$\overline{e}$	Vector of predicted Errors
d	Prediction error
A	Dynamic matrix
$\hat{y}_v$	Predicted process variable for process variable r
u	Controller output variable
V	number of measured process variables
S	number of controller outputs

Greek Symbols

β_1	Valve coefficient of MV_1
β_2	Valve coefficient of MV_2
β_{12}	Valve coefficient of MV_{12}
$\Delta\overline{u}$	Vector of controller output moves to be determined
$\Lambda^T\Lambda$	Matrix of move suppression coefficients
$\Gamma^T\Gamma$	Matrix of controlled variable weights
λ	Controlled variable weights
γ	Move suppression weights

Abbreviations

MIMO	Multi Input Multi Output
TCTILS	Two Conical Tank Interacting Level System

TITO	Two Input Two Output
DMC	Dynamic Matrix Controller
MPC	Model Predictive Controller
PID	Proportional Integral Derivative
FOPDT	First Order plus Dead Time
DLO	Design Level of Operation
ISE	Integral Square Error
IAE	Integral Absolute Error
ITAE	Integral Time Absolute Error

MPC algorithm used in the process industry for the past two decades. A major part of DMC s appeal in industry from the use of a linear convolution model derived from step-response of the process and a quadratic performance objective function. The objective function is minimized over a prediction horizon to compute the optimal controller output moves as a least squares problem.

Conical tanks find wide applications in process industries, namely hydrometallurgical industries, food processing industries, concrete mixing industries, sewage water treatment industries and wastewater treatment industries. Their shape contributes to better drainage of solid mixtures, slurries and viscous liquids. In this work, Two Conical Tank Interacting Level System (TCTILS) is taken up for study which is TITO system having interaction and input-output constraints. Control of TCTILS presents a challenging problem due to its non-linearity and dynamic interaction. As the operating point changes, the parameters of TCTILS change due to constantly changing cross sectional area.

Advanced control schemes such as fuzzy control scheme [4], Neuro based model reference control scheme [5] and optimal control scheme [6] have been already attempted on single conical tank. A non adaptive decentralized PID control scheme with single decoupler [7] has been also attempted on TCTILS. Even with the introduction of above mentioned powerful nonlinear control strategies, the control scheme presented in this paper remains an attractive one in process industries.

The paper is organized as follows: The TCTILS process considered for simulation study has been discussed in Section 2. In Section 3, multivariable Dynamic Matrix Control is discussed. The implementation of DMC for TCTILS process is discussed in section 4. The simulation studies are discussed in Section 5. Finally, conclusions are discussed in Section 5.

2. Process Description

The TCTILS as shown in Fig. 1 is based on the two-tank benchmark problem that had been used by a number of researchers. TCTILS consists of two identical conical tanks (TANK1 and TANK2), two independent pumps (PUMP1 and PUMP2) that deliver the liquid flows F_{IN1} and F_{IN2} to TANK1 and TANK2 through the two control valves CV_1 and CV_2 respectively. These two tanks are interconnected at the bottom through a manually controlled valve, MV_{12} with a valve co-efficient β_{12}. F_{OUT1} and F_{OUT2} are the two output flows from TANK1 and TANK2 through manual control valves MV_1 and MV_2 with valve coefficients β_1 and β_2 respectively. The β_{12}, β_1 and β_2 are adjustable coefficients representing the resistance of the respective valves opening orifice. The valve coefficient β_i may be evaluated using (1),

$$\beta_i = C_i a_i \sqrt{2g} \qquad (i = 1, 2, 12) \tag{1}$$

where C_i (i = 1, 2,12) are the cross sectional area of valves and a_i is the discharge coefficient, which is assumed as unity.

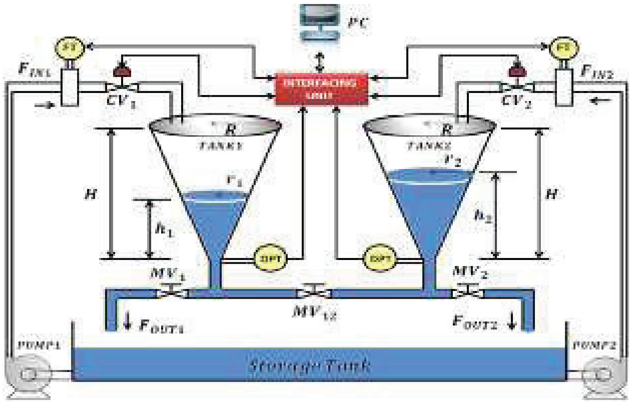


Fig.1.Schematic of TCTILS

The operating parameters of TCTILS are shown in Table 1. In TCTILS, h_1 & h_2 are considered as output variables and F_{IN1} & F_{IN2} are considered as respective manipulated variables. The mathematical model for TCTILS is given by (1) and (2).

$$\frac{dh_1}{dt} = \left[\frac{F_{IN1} - \frac{1}{3} h_1 \frac{dA(h_1)}{dt} - \beta_1 \sqrt{h_1} - sign(h_1 - h_2)\beta_{12}\sqrt{|h_1 - h_2|}}{\frac{1}{3}\pi R^2 \frac{h_1^2}{H^2}} \right] \tag{2}$$

$$\frac{dh_2}{dt} = \left[\frac{F_{IN2} - \frac{1}{3} h_2 \frac{dA(h_2)}{dt} - \beta_2 \sqrt{h_2} + sign(h_1 - h_2)\beta_{12}\sqrt{|h_1 - h_2|}}{\frac{1}{3}\pi R^2 \frac{h_2^2}{H^2}} \right] \tag{3}$$

Table 1. Operating Parameters of TCTILS

Parameter	Description	Value
R	Top radius of conical tank	19.25 cm
H	Maximum height of TANK1 & TANK2	73 cm
F_{IN1} & F_{IN2}	Maximum Inflow to TANK1& TANK2	138.89 cm^3/s
β_1	Valve co-efficient of MV_1	50 cm^2 / s
β_{12}	Valve co-efficient of MV_{12}	35 cm^2 / s
β_2	Valve co-efficient of MV_2	50 cm^2 / s

3. Multivariable Dynamic Matrix Control

Multivariable DMC has been discussed extensively by past researchers [8-9] and is summarized here for the convenience of the reader. For a system with S controller outputs and V measured process variables, the multivariable DMC quadratic performance objective function has the form [10]

$$\min_{\Delta\bar{u}} J = [\bar{e} - A\Delta\bar{u}]^T \Gamma^T \Gamma[\bar{e} - A\Delta\bar{u}] + [\Delta\bar{u}]^T \Lambda^T \Lambda[\Delta\bar{u}] \tag{4}$$

subject to

$$\begin{aligned} &\hat{y}_{v,\min} \le \hat{y}_v \le \hat{y}_{v,\max}, \\ &\Delta\bar{u}_{s,\min} \le \Delta\bar{u}_s \le \Delta\bar{u}_{s,\max}, \\ &\bar{u}_{s,\min} \le \bar{u}_s \le \bar{u}_{s,\max} \end{aligned} \tag{5}$$

A closed form solution to the multivariable DMC performance objective (Eq. (4)) results in the unconstrained multivariable DMC control law [10]:

$$\Delta\bar{u} = (A^T\Gamma^T\Gamma A + \Lambda^T\Lambda)^{-1} A^T\Gamma^T\Gamma\bar{e} \tag{6}$$

Here, A is the multivariable dynamic matrix formed from unit step response coefficients of each controller output to measured process variable pair; $\bar{e}$ is the vector of predicted errors for the R measured process variables over the next P sampling instants (prediction horizon); $\Delta\bar{u}$ is the vector of controller output changes for the S controller output computed for the next M sampling instants (control horizon); $\hat{y}_v$ is the predicted process variable profile for the r^{th} measured process variable over the next P sampling instances; $\Gamma^T\Gamma$ is the matrix of controlled variable weights and $\Lambda^T\Lambda$ is the matrix of move suppression coefficients.

$\Lambda^T\Lambda$ is a square diagonal matrix of dimensions (MS×MS). The leading diagonal elements of the i^{th} (M×M) matrix block along the diagonal of $\Lambda^T\Lambda$ are λ_i^2. All off-diagonal elements are zero. Hence, in the multivariable DMC control law, the suppression coefficients that are added to the leading diagonal of the system matrix, $(A^T\Gamma^T\Gamma A)$, are λ_i^2. (i=1,2,….,S). Similarly, the (PR × PR) matrix of controlled variable weights, $\Gamma^T\Gamma$, has the leading diagonal elements as γ_i^2 (i=1,2,……,R). Again, all off-diagonal elements are zero.

The implementation of DMC involves using step response coefficients to predict the future process variable behaviour, $\hat{y}_V(k+1)$ over the prediction horizon. This profile is corrected by adding to it estimates of the disturbance. The disturbance estimates are calculated as the difference between the current measurement of the process variable and the current value of the predicted process variable at the present sample time. The disturbance estimate is assumed constant over the prediction horizon. Then, (4) or (6) is solved on-line to determine the optimal values of the controller output moves. Only the first element of the vector is implemented and the entire procedure is repeated at the next sampling instance.

4. DMC implementation for TCTILS

The DMC controller is designed for TCTILS using the tuning rules given by Shridhar and Cooper [11]. The tuning parameters and the step response coefficients are calculated offline prior to the start-up of the DMC controller and remain constant during operation. Step response coefficients for the internal DMC process model do not use an FOPDT approximation. Rather, actual process data is employed as is typical for DMC. The process data is generated by introducing a positive step in one controller output with the process at steady state and all the controllers in manual mode. In addition, all other controller output variables must remain constant. From the instant the step change is made, the response of each process variable is recorded as it evolves and settles at a new steady state. For a step in the controller output of arbitrary size, the response data is normalized by dividing through by the size of the controller output step to yield the unit step response. This is performed for each controller output to measured process variable pair, and it is necessary to make the controller output step large enough such that noise in the process variable measurement does not mask the true process behaviour.

For a TCTILS, the multivariable dynamic matrix, A, is formulated using the first P step response coefficients:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}_{2P \times 2M} \tag{7}$$

where A_{11} is constructed from the step response of process variable 1 (h_1) obtained by a step change in controller output 1 (F_{IN1}). A_{ii} is given by

$$A_{ij} = \begin{bmatrix} a_{ij,1} & 0 & 0 & \cdots & 0 \\ a_{ij,2} & a_{ij,1} & 0 & & 0 \\ a_{ij,3} & a_{ij,2} & a_{ij,1} & \ddots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a_{ij,M} & a_{ij,M-1} & a_{ij,M-2} & \cdots & a_{ij,1} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{ij,P} & a_{ij,P-1} & a_{ij,P-2} & \cdots & a_{ij,P-M+1} \end{bmatrix}_{P \times M} \tag{8}$$

Using (4) and (5) for constrained DMC or (6) for unconstrained systems, a vector of (SM) controller output moves is computed over the control horizon:

$$\Delta \overline{u} = \begin{bmatrix} \Delta u_1(n) \\ \Delta u_1(n+1) \\ \Delta u_1(n+2) \\ \vdots \\ \Delta u_1(n+M-1) \\ \Delta u_2(n) \\ \Delta u_2(n+1) \\ \Delta u_2(n+2) \\ \vdots \\ \Delta u_2(n+M-1) \end{bmatrix}_{2M \times 1} \tag{9}$$

where $\Delta u_1(n)$ is the move implemented for F_{IN1} and $\Delta u_2(n)$ is the move implemented for controller output 2 (F_{IN2}).

The tuning parameters and the DLO parameters used to obtain step response coefficients for TCTILS are calculated offline and are reported in Table 2. The DMC control strategy implemented for TCTILS is shown in Fig. 2.

Table 2. DMC tuning parameters for TCTILS

Tuning Parameters	Value
Design Level of Operation to obtain Step response Model	
(i) Input flows F_{IN1} & F_{IN2} (LPH)	50, 25
(ii) Heights h_1 and h_2 (cm)	0.4039, 0.101
Prediction Horizon (P)	85
Control Horizon (M)	19
Model Horizon (N)	2750
Controlled variable weight for h_1 & h_2 (γ_1 , γ_2)	20, 20
Controller output weight for F_{IN1} & F_{IN2} (λ_1 , λ_2)	3, 3

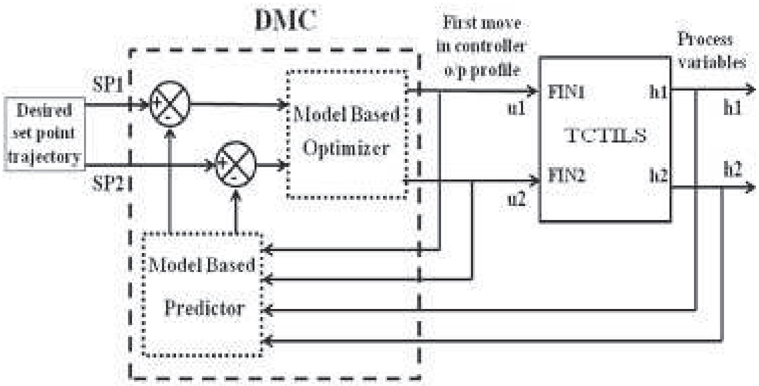


Fig. 2. DMC control scheme for TCTILS

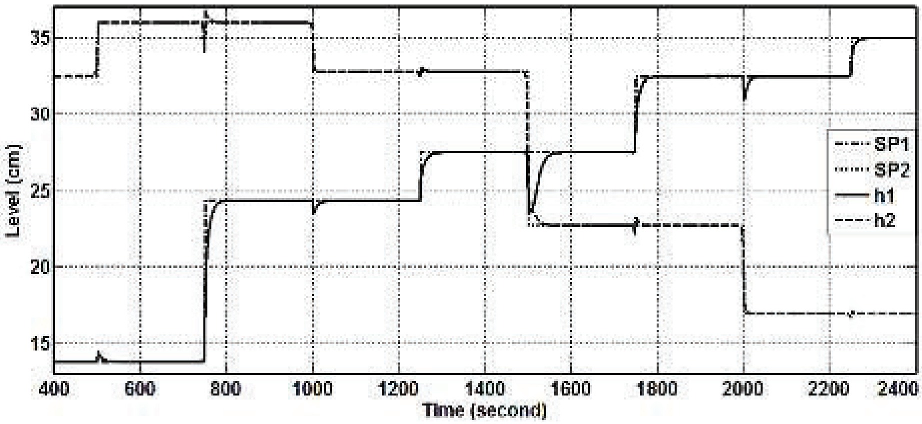
4. Simulation Studies

Simulation studies are carried out on the TCTILS. The process is simulated using the nonlinear first principle model (Equation (2) and (3)). The output variables are computed by solving the nonlinear differential equations using differential equation solver in Matlab R2009a. The DMC algorithm is implemented in Matlab R2009a.

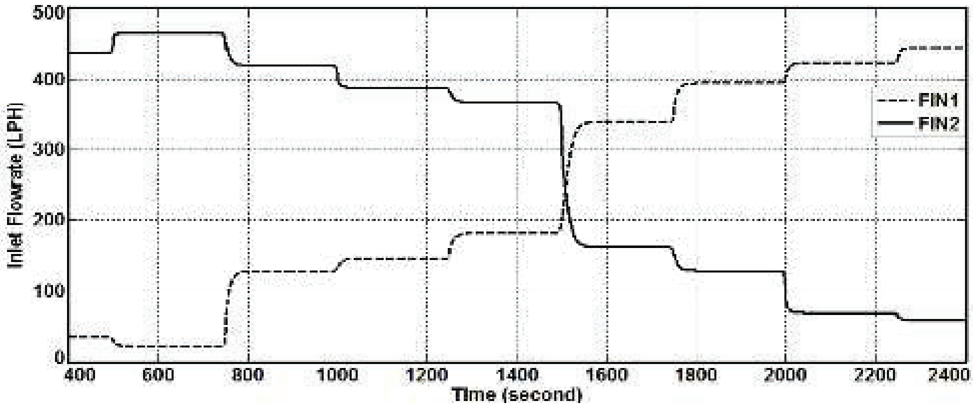
4.1 Servo performance

The setpoint variations (Fig. 3(a)) are introduced for assessing the tracking capability of the DMC. From the response, it can be inferred that, the DMC is able to maintain the tank levels h_1 and h_2 at the respective setpoints. The performance indices for the DMC are reported in Table 3. The variations in the controller outputs are smooth and are presented in Fig. 3(b). The observations (qualitative) of the above simulation study are as follows:

DMC is able to maintain the h_1 and h_2 at the respective setpoints. The performances of DMC at all the operating points are found to be better, as there is no overshoot, less interaction and settles to the setpoint faster.



(a)



(b)

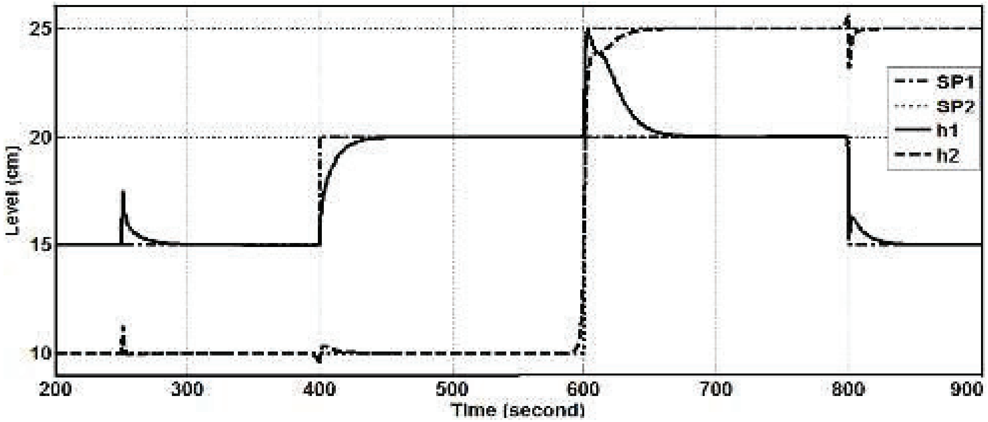
Fig.3. Servo response of TCTILS with Dynamic Matrix Controller (a) Process output (b) Control signal

4.2 Servo-regulatory performance

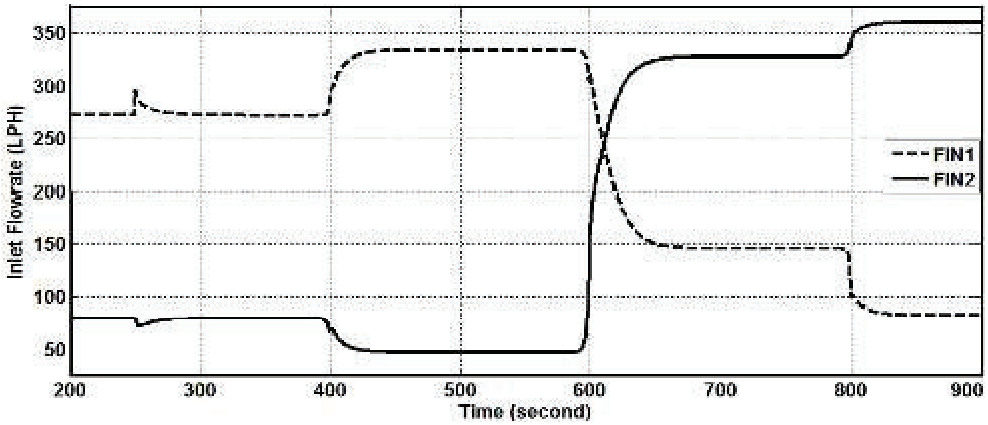
Simulation studies are carried out to demonstrate the disturbance rejection capability of the DMC at nominal and at shifted operating points. A step disturbance of magnitude 25 LPH is introduced in F_{IN1} at time = 250 second and is then removed at time = 800 second. The performance indices are computed for DMC and are reported in Table 3. The following observations are drawn from simulation studies:

- From simulation time = 250 second to 350 second the DMC is able to reject the disturbance quickly and bring the levels h_1 and h_2 back to the nominal value of the respective setpoints as shown in Fig. 4(a). This clearly demonstrates that the controllers are able to reject the disturbance at the nominal operating point.

- With the disturbance being persistent, a step change in the setpoints h_1 and h_2 are introduced at time = 400 and 600 second respectively. It can be noted that the DMC is able to maintain the levels h_1 and h_2 at the respective setpoints.
- At simulation time = 800 second, simultaneously a step change in the setpoint h_1 is introduced and the disturbance is removed. This part of the simulation demonstrates that the DMC is able to reject the disturbance as well as maintain the process variables at the respective setpoints.



(a)



(b)

Fig.4. Servo- regulatory response of TCTILS with DMC (a) Process output (b) Control signal

Table 3. Performance indices in the presence of setpoint change and load change in F_{IN1}

Performance Indices	Servo Performance		Servo with Regulatory Performance	
	h1	h2	h1	h2
ISE	709.6905	1.4466e+03	997.7105	522.3130
IAE	290.5889	162.5097	282.7066	115.9304
ITAE	3.5144e+005	1.3272e+005	1.8063e+005	5.0179e+004

5. Conclusions

In this paper, the authors have suggested a simple and straightforward procedure for designing a Dynamic Matrix Control (DMC) scheme for TCTILS process, which exhibits significant nonlinear dynamics and dynamic interaction. From the extensive simulation studies, it is concluded that the DMC controller has good setpoint tracking, disturbance rejection capabilities at nominal and shifted operated points. Further, it can be concluded that the DMC controller helps to produce response with no overshoot, less interaction and settles to the setpoint faster in the entire operating region.

Acknowledgements

The authors acknowledge the financial support of AICTE, New Delhi through the project titled IMPLEMENTATION OF ADVANCED CONTROLLERS FOR INTERACTING NON LINEAR SYSTEM vide File no : F.N : 8023 / RID / BOR / MOD – 339 / 2007-2008

References

[1] Smith, Townsend S, Irwin G. Non-linear model based predictive control using multiple local models. In: Kouvaritakis B, Cannon (Eds.) M. Non-linear predictive control: Theory and practice, IEE Control Engineering Series, IEE, London; 2001, 61(11), 47–56.

[2] Froisy J B. Model predictive control: Past, present and future. ISA Transaction 1994; 33, 235–243.

[3] Cutler C R., Ramaker D L. Dynamic matrix control—a computer control algorithm. Proceedings of the JACC, San Francisco,CA, 1980.

[4] Anandanatarajan R, Chidambaram M, Jayasingh. T. Limitations of a PI controller for a first order non-linear process with dead time, ISA Transactions, 2006,45, 185-200.

[5] Bhuvaneswari N S,Uma G , Rangaswamy T R. Neuro based Model Reference Adaptive control of a conical tank level process, Control and Intelligent Systems, 2008, 36(1), 1-9.

[6] Bhuvaneswari N S, Uma G, Rangaswamy T R. Adaptive and Optimal Control of a Non-Linear process using Intelligent Controllers, Applied soft computing, 2008, 9(1), 182-190.

[7] Ravi V R, Thyagarajan T. A De-centralised PID Controller for Interacting Non linear Systems, Proceeding of **IEEE** 2011 International Conference on Emerging trends in Electrical and Computer Technology, 2011, 297-302.

[8] Cutler, C. R., & Ramaker, D. L. Dynamic matrix control—a computer control algorithm. Proceedings of the JACC, 1980, San Francisco, CA.

[9] Marchetti, J. L., Mellichamp, D. A., & Seborg, D. E. Predictive control based on discrete convolution models. Industrial and Engineering Chemistry, Process Design and Development, 1983,22, 488–495.

[10] Garcıa, C. E., & Morshedi, A. M. (1986). Quadratic programming solution ofdynamic matrix control (QDMC). Chemical Engineering Communications, 46, 73–87.

[11] Shridhar R, Cooper D J. A tuning strategy for unconstrained multivariable model predictive control. Industrial and Engineering Chemistry Research, 1998, 37, 4003–4016.